

Coherent Muon-Electron Conversion in the Dualized Standard Model

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Abstract

Muon-electron conversion in nuclei is considered in the framework of the Dualized Standard Model. The ratio $B_{\mu-e}$ of the conversion rate to the total muon capture rate is derived, and computed for several nuclei in a parameter-free calculation using parameters previously determined in different physical contexts. The values obtained all lie within the present experimental bounds, but some are so close as to seem readily accessible to experiments already being planned. Similar considerations are applied also to muon-electron conversion in muonium but give rates many orders of magnitude below the present experiment limit.

In this note we present some results of the Dualized Standard Model (DSM) for coherent muon-electron conversion in the nuclear elastic (ground state to ground state) process:

$$\mu^- + (Z, N) \rightarrow e^- + (Z, N). \quad (1)$$

This process entails a change of flavor number and is forbidden in the conventional version of the Standard Model. It is allowed however in many extensions of Standard Model considered in the literature with so-called “horizontal” symmetries linking the different generations. (For a general discussion of the problem see [1]). For this reason, it is widely accepted that precision experiments on muon-electron conversion are an excellent framework to test theories beyond the conventional Standard Model.

Present experiments give the best limits on the ratio of the conversion rate to the muon capture rate as [2]:

$$\begin{aligned} B_{\mu-e}^{Pb} &\leq 4.6 \times 10^{-11} \\ B_{\mu-e}^{Ti} &\leq 4.3 \times 10^{-12} \end{aligned} \quad (2)$$

which put stringent limits on flavor-changing currents. Furthermore, experimental sensitivity is expected to be improved by three or four orders of magnitude by the planned experiment at PSI [3] using a ^{48}Ti target, and the MECO experiment [4] at Brookhaven, using a ^{27}Al target. They will thus afford very valuable tests for extensions beyond the conventional Standard Model.

However, most extensions of the Standard Model (SUSY, GUT’s...) involve ingredients outside the theoretical framework of the Standard Model. Their predictions on muon-conversion processes would thus generally depend on various “external” parameters (symmetry breaking scales, mixing matrices...) which are not given by the theory but have to be determined from experiment on non-standard effects, so that their predictive power is in general limited. In contrast, the DSM scheme suggested in Ref. [5] relies on an earlier result that within the Standard Model itself there is already a dual color group which can play the role of the horizontal symmetry of generations. Most of the parameters in the scheme are thus either already given by the theory or else determined by fitting standard quantities such as masses and mixing angles, so that actual predictions for non-standard processes can now be given in a quite unambiguous manner.

Our aim in this paper is to examine the predictions of the DSM scheme on muon-conversion processes using the formalism and techniques developed previously for an analysis on flavor-changing neutral currents effects in meson mass differences and rare meson decays [6]. Muon conversion rates are then obtained, which turn out to be quite close to the present experimental limits and should thus be testable by experiment in the very near future.

We begin with a brief outline of the basic tenets of the DSM scheme. Based on a non-Abelian generalization of electric-magnetic duality [7] and a result on

confinement of 't Hooft's [8], the dualized version of the Standard Model has been constructed [5] which offers an explanation for the existence of just three generations of fermions as a broken local dual color gauge symmetry, and also of Higgs fields as frame vectors in dual color space. This dual color $\widetilde{SU}(3)$ group which is identified with the generation symmetry, is spontaneously broken in such a way that, at the one-loop level of dual boson-exchange, the (real) mixing matrices and masses can be calculated for both quarks and leptons. The results obtained are in good agreement with present experiment. A full account of the whole process is given in [9, 10, 11, 12] where it was shown that with only three free parameters (the two ratios between the three dual Higgs v.e.v.'s plus a common Yukawa coupling) one is able to reproduce to an acceptable level some 14 of the "fundamental" parameters of the Standard Model.

We note in particular the following features of the DSM scheme which are of special relevance to the problem at hand. First, by virtue of the Dirac quantization condition, no unknown gauge couplings appear in the DSM. The gauge couplings \tilde{g}_i of the dual groups are related to those of the "direct" groups g_i [13], namely the ordinary color and electroweak gauge couplings routinely measured in present day experiments:

$$g_{3(2)}\tilde{g}_{3(2)} = 4\pi, \quad g_1\tilde{g}_1 = 2\pi.$$

In other words, the coupling strengths \tilde{g}_3 and \tilde{g}_1 of the dual gauge bosons, which we shall need in what follows for considerations of muon conversion, are derivable from respectively the usual strong coupling constant g_3 and the coupling of weak hypercharge g_1 .

Secondly, the branching of these couplings \tilde{g}_i into the various physical fermion states are given by the orientations of these physical states in generation or dual color space. If we denote by ψ_L^A the left-handed fermion fields of type A , the relation between the *physical* and *gauge* basis in generation space is given by a unitary matrix S^A :

$$\psi_{gauge,L}^A = S^A \psi_{physical,L}^A \quad (3)$$

where the index A runs over the four types of fermions U, D, L (charged leptons) and N (neutrinos). These orientation matrices S^A were already determined as by-products in the calculation of fermion mixing matrices [9, 10] by fitting the free parameters of the model to the masses of the higher generation fermions and to the Cabibbo angle. Those matrices for quarks and leptons which are relevant for later development in this paper are given in [6]. We note that in terms of S^A , the CKM matrix for leptons (quarks) is given simply as $V_{CKM}^{lepton} = (S^N)^\dagger S^L$ ($V_{CKM}^{quark} = (S^U)^\dagger S^D$), and the result on the mixing matrices of [9, 10, 11] was found to be in excellent agreement with present experimental values.

Thirdly, the fit done in [9] quoted above also gave a hierarchical relation between the v.e.v.'s of the Higgs bosons responsible for breaking the dual color or generation symmetry, which implies in turn that in the tree-level spectrum found

for the dual color gauge bosons, one particular state has a much lower mass than the rest, so that the calculation of the low energy effective Lagrangian relevant for one-dual gauge boson exchange becomes quite simple, being dominated by just the exchange of this one boson. We give here the expression for the full effective Lagrangian [6],

$$L_{eff} = \frac{1}{2(\zeta z)^2} \sum_{A,B} f_{\alpha,\beta;\alpha',\beta'}^{A,B} (J_A^{\mu\dagger})^{\alpha,\beta} (J_{\mu,B})^{\alpha',\beta'}, \quad (4)$$

with currents of the usual $V - A$ form:

$$(J_\mu^A)_{\alpha,\beta} = \bar{\psi}_{L,\alpha}^A \gamma_\mu \psi_{L,\beta}^A,$$

where Greek indices run over flavors, and the group factor, in the present case in which only the lightest gauge boson is relevant, reduces to the following particularly simple combination of the rotating matrices (3):

$$f_{\alpha,\beta;\alpha',\beta'}^{A,B} = S_{3,\alpha}^{A*} S_{3,\beta}^A S_{3,\alpha'}^{B*} S_{3,\beta'}^B \quad (5)$$

The structure of this L_{eff} is self explanatory in the sense that it links together fermions of equal or different flavors (A and B).

Finally, the only remaining unknown among the quantities required which appeared in (4) is the mass scale ζz of the breaking of dual color symmetry. This parameter was not constrained by the calculation of mixing matrices in [9, 10, 11] but are bounded by other considerations [6, 14] as follows. By studying the effects of dual color symmetry breaking on FCNC meson decay and on the mass differences of conjugate neutral meson pairs, one finds that the mass difference $K_L - K_S$ is the most restrictive, giving a lower bound for the scale ζz of the order of a few hundred TeV [6]. On the other hand, accepting the suggestion [14] that those rare air shower events with energies beyond the GZK cut-off [15] are produced by neutrinos acquiring strong interactions through the exchange of dual gauge bosons, one can give a rough upper bound to the scale ζz also of the order of a few hundred TeV [6]. All in all, our best guess for this scale ζz , which in the DSM scheme is the mass of the lightest dual Higgs boson, is in the range 300 – 500 TeV. This is much lower than the scales appearing in other models beyond the SM such as GUT's. Notice, however, that these quoted limits for ζz are merely rough estimates, especially the upper bound which was deduced from the neutrino explanation of post-GZK air showers by only qualitative arguments and awaits a detailed analysis when more data become available. Nevertheless, these estimates will give one some indications for the sort of values to be expected.

With these tools in hand and all parameters fixed, we now turn to the problem of muon-electron conversion in nuclei. We shall first extract from (4) the piece relevant for these transitions. The result will be just the product of two currents, firstly a leptonic piece made out of single left-handed lepton fields with different

flavors to describe the muon-electron conversion, and secondly a flavor-conserving hadronic current corresponding to the fact that the initial and final nuclei have the same number of neutrons and protons. This is multiplied by an effective coupling strength which is essentially the inverse square of the scale ζz times a group factor $f_{\alpha,\beta;\alpha',\beta'}^{A,B}$ coming from the rotation between gauge and physical states in generation or dual color space (5). Explicitly, at the quark and lepton level, the relevant piece of the effective Lagrangian for the conversion processes of interest in this paper is as follows¹:

$$L_{eff}^{L,Q} = \frac{1}{(\zeta z)^2} (e_L \gamma^\mu \bar{\mu}_L) \left\{ f_{3,2;3,3}^{L,U} \bar{u}_L \gamma_\mu u_L + f_{3,2;3,3}^{L,D} \bar{d}_L \gamma_\mu d_L \right\}, \quad (6)$$

This embodies all the elementary information we need for computing the conversion probability in the one dual gauge boson exchange approximation.

The next step is to find the effective Lagrangian appropriate for the nuclear process (1), i.e. to pass from the Lagrangian given in terms of the elementary quark fields to a Lagrangian in terms of neutrons and protons. Usually, the passage from quarks to nucleons is effected by using the Lorentz invariant form factors depending on the momentum transfer (q^2), which provide a phenomenological parametrization of the non-perturbative structure of the nucleons as quark bound states, and can be largely determined through experimental observations and symmetry considerations. In the present case, the momentum transfer is very small compared to any other mass scale involved in the process, so that one can ignore the terms which are suppressed by factors of (q^2) over the nucleus mass, as well as the dependence on (q^2). Hence one can just keep the contributions of the vector and axial vector form factors $G_{V,A}(q^2)$ taken at $q^2 = 0$, which leads to the following approximation for the nuclear currents:

$$\begin{aligned} \langle \phi | \bar{q} \gamma_\mu q | \phi \rangle &= G_V^{q,N} \bar{\phi} \gamma_\mu \phi, \\ \langle \phi | \bar{q} \gamma_\mu \gamma_5 q | \phi \rangle &= G_A^{q,N} \bar{\phi} \gamma_\mu \gamma_5 \phi, \end{aligned}$$

where ϕ represents the wave function of a single nucleon N , which can be either a proton p or a neutron n .

In the limit of exact isospin symmetry, the vector piece of the nucleon form factors is given by:

$$\begin{aligned} G_V^{u,n} &= G_V^{d,p} = G_V^d = 1, \\ G_V^{u,p} &= G_V^{d,n} = G_V^u = 2. \end{aligned}$$

The last equality in both cases comes from considering a coherent contribution to the scattering process from the valence quarks inside the nucleon. The same argument cannot be directly applied to the axial-vector piece but, for practical

¹Note that the ordering convention of the generation indices here is from high to low mass.

reasons this is irrelevant to us: being dependent on the nuclear spin (in the non-relativistic limit) it is incoherent from the point of view of the nucleons. Only nucleons outside closed shells contribute to this piece and it is much smaller than the vectorial, coherent, contribution². Thus, for our purpose we shall just parametrize this contribution in a generic way by the axial vector form factor $g_A^{(N)}$. The effective Lagrangian in terms of proton and neutron fields then reads as:

$$L_{eff}^{L,Q} = \frac{1}{(\zeta z)^2} (e_L \gamma^\mu \bar{\mu}_L) \left\{ \left(2f_{3,2;3,3}^{L,U} + f_{3,2;3,3}^{L,D} \right) \left(\bar{p} \gamma_\mu \frac{1 - g_A^{(p)} \gamma_5}{2} p \right) \right. \\ \left. + \left(f_{3,2;3,3}^{L,U} + 2f_{3,2;3,3}^{L,D} \right) \left(\bar{n} \gamma_\mu \frac{1 - g_A^{(n)} \gamma_5}{2} n \right) \right\}, \quad (7)$$

where p and n correspond to the spinor fields for respectively protons and neutrons.

It is sometimes useful in nuclear processes to use the isospin formalism and write the fields in an isospin doublet (N) . For completeness, we give the form of (7) also in this notation:

$$L_{eff}^{L,N} = \frac{1}{(\zeta z)^2} (e_L \gamma^\mu \bar{\mu}_L) \left\{ \frac{3}{2} \left(f_{3,2;3,3}^{L,U} + f_{3,2;3,3}^{L,D} \right) \left(\bar{N} \gamma_\mu \frac{1 - g_A^{(p)'} \gamma_5}{2} N \right) \right. \\ \left. + \frac{1}{2} \left(f_{3,2;3,3}^{L,U} - f_{3,2;3,3}^{L,D} \right) \left(\bar{N} \gamma_\mu \frac{1 - g_A^{(n)'} \gamma_5}{2} \tau_3 N \right) \right\},$$

where $g_A^{(N)'}$ is an averaged axial form factor including both $g_A^{(p)}$ and $g_A^{(n)}$ [16]. Its explicit form is not important here.

Next, using the non-relativistic approximation for the nuclear motion, which allows us to consider only the large component of the nucleon wave function (notice that the small component is $O(\vec{p}/M)$ for a nucleon of mass M and momentum \vec{p}), we can perform the following substitution in the Dirac γ matrices: $\gamma^0 \rightarrow I$, $\vec{\gamma} \sim \gamma^0 \gamma^5 \sim \vec{p}/M \rightarrow 0$, $\vec{\gamma} \gamma^5 \rightarrow \vec{\sigma}$ when taken between hadron states. The effective Lagrangian can then be expressed in the following form:

$$L_{eff}^{L,N} = \frac{1}{2(\zeta z)^2} \left\{ (e_L^+ \mu_L) \left[\frac{3}{2} \left(f_{3,2;3,3}^{L,U} + f_{3,2;3,3}^{L,D} \right) (N^+ N) + \right. \right. \\ \left. \left. + \frac{1}{2} \left(f_{3,2;3,3}^{L,U} - f_{3,2;3,3}^{L,D} \right) (N^+ \tau_3 N) \right] - \right. \\ \left. - (\bar{e}_L \vec{\gamma} \mu_L) \left[g_A \frac{3}{2} \left(f_{3,2;3,3}^{L,U} + f_{3,2;3,3}^{L,D} \right) (N^+ \vec{\sigma} N) + \right. \right. \\ \left. \left. + g_A' \frac{1}{2} \left(f_{3,2;3,3}^{L,U} - f_{3,2;3,3}^{L,D} \right) (N^+ \vec{\sigma} \tau_3 N) \right] \right\},$$

²For a detailed discussion of the axial-vector piece, see e.g. [16].

Further, as already mentioned above, the $\vec{\sigma}$ dependent term is proportional to the spin of the nucleus which, in turn, depends on the number of nucleons outside closed shells. In middle and heavy nuclei, this number is much smaller than the atomic weight A . On the other hand the τ_3 terms can be interpreted as the isospin density which is comparable to $(Z - N)\rho(x)$, $\rho(x)$ being the average density for protons and neutrons inside the nucleus. Compared to the atomic weight, this term is again negligible for middle nuclei where Z and N are comparable. Hence, for the nuclei we shall be most interested in, the only remaining term is N^+N which in the position configuration is directly proportional to the nuclear density. Normalizing ρ to unity, we can then make the substitution:

$$N^+N \rightarrow A\rho(x).$$

As a result, we get the final form of the effective Lagrangian relevant to the muon-electron conversion process, which simply reads as:

$$L_{eff}^{L,N} = \frac{1}{(\zeta z)^2} Q \rho(x) e_L^+ \mu_L, \quad (8)$$

where

$$Q = \frac{3A}{4} \left(f_{3,2;3,3}^{L,U} + f_{3,2;3,3}^{L,D} \right). \quad (9)$$

It is interesting to note in (8) that all the information which depends on the flavor-violation scheme, in our case the DSM, is contained just in the factor $Q/(\zeta z)^2$. The remaining part in (8) depends only on the (approximated) nuclear dynamics. This factor $Q/(\zeta z)^2$ here thus plays the role of a *flavor-violating charge* in giving the flavor-violating coupling of the lepton to the nucleus in the one dual gauge boson exchange approximation. One can thus translate the result of (8) directly to any other flavor-changing model mediated by one boson exchange just by the substitution of the *flavor-violating charge* appropriate to that model.

In the expression (9) for Q , we have assumed that the numbers of protons (Z) and neutrons (N) are roughly the same. More generally, we can rewrite the expression keeping track of these numbers as follows:

$$Q = \frac{1}{2} \left[Z \left(2f_{3,2;3,3}^{L,U} + f_{3,2;3,3}^{L,D} \right) + N \left(f_{3,2;3,3}^{L,U} + 2f_{3,2;3,3}^{L,D} \right) \right], \quad (10)$$

where for simplicity, we have taken similar proton and neutron densities inside the nucleus. In general, it will depend weakly on the nuclear parameters, essentially through the ratio $\frac{A-2Z}{A}$.

Substituting next the effective Lagrangian (8) into the expression calculated from the scattering process, one obtains for the conversion rate:

$$\Gamma_{conv.} = \frac{p_e E_e}{2\pi} \frac{1}{(\zeta z)^4} Q^2 |M(q^2)|^2. \quad (11)$$

where p_e and E_e are the momentum and energy of the electron which, in the present low energy case, are both of the order of the muon mass. In the formula (11), we have taken the non-relativistic limit for the motion of the muon in the muonic atom and in this way we can factorize the large component of the muon wave function, $\phi_\mu(x)$. $M(q^2)$ is then the Fourier transform of the nucleus density modulated by the muon wave function:

$$M(q^2) = \int d^3x \rho(x) e^{-i\vec{q}\vec{x}} \phi_\mu(x). \quad (12)$$

Now, for nuclei with $A \leq 100$ it is customary in μ -capture analysis to take an average value for the muon wave function inside the nucleus and parametrize the nuclear size effects by the form factor in such a way that eq. (12) becomes

$$|M(q^2)|^2 = \frac{\alpha^3 m_\mu^3}{\pi} \frac{Z_{eff}^4}{Z} |F(q^2)|^2, \quad (13)$$

where Z_{eff} has been determined in the literature [18, 19] and is given by essentially the square of the muon wave function averaged over the nucleus, and $F(q^2)$ is the nuclear form factor which can be measured for instance in electron scattering. For our purposes we can approximate $F(q^2)$ by a dipole form factor which reproduces the mean squared radius of the nucleus, thus:

$$F(q^2) = \frac{1}{1 - \frac{q^2 \langle r^2 \rangle}{6}} \quad (14)$$

where the mean squared radius is approximately given by $\langle r^2 \rangle^{1/2} = 1.2A^{1/3} \text{ fm}$. This approximation works very well for nuclei with $A \leq 100$ although it tends to overestimate the form factor for heavy nuclei. All in all, we considered our approximations in nuclear physics to be valid at the level of a few percent in nuclei with $A \leq 100$ (such as *Al*, *S* and *Ti*), whereas for heavy nuclei (e.g *Pb*) they will give only the rough order of magnitude.

What is usually compared with experiment is not the absolute value of the conversion rate but the ratio of this rate to the total muon capture rate, since this latter quantity is experimentally determined with very good precision [17]. However for the rough orders of magnitude that we are interested in, we could also take the theoretical expression for the muon capture rate which has the virtue of eliminating in the ratio most of the nuclear dependence and can be applied to a wider range of elements where no accurate measurement is yet available. The standard semi-empirical expression for the nuclear muon capture rate is: [20]:

$$\Gamma_{cap.} = \frac{G_F^2 m_\mu^5 \alpha^3}{2\pi^2} (g_V^2 + 3g_A^2) Z_{eff}^4 \left(1 - \delta \frac{N}{2A}\right) \quad (15)$$

where we can see the contribution of the Fermi and Gamow-Teller transition. The last bracket, which is important for heavy nuclei, appears by virtue of the

<i>Element</i>	Z_{eff}	$\Gamma_{cap.}^{Exp.}(s^{-1})$	$B_{\mu-e}^{Theor.}$	$B_{\mu-e}^{Exp.limit}$
$^{27}Al_{13}$	11.48	0.66×10^6	2.8×10^{-12}	<i>n.a.</i>
$^{32}S_{16}$	13.64	1.34×10^6	2.6×10^{-12}	7×10^{-11}
$^{48}Ti_{22}$	17.38	2.6×10^6	4.6×10^{-12}	4.3×10^{-12}
$^{207}Pb_{82}$	34.18	13.3×10^6	5.3×10^{-12}	4.6×10^{-12}

Table 1: Theoretical estimates for the ratio of the $\mu - e$ conversion rate to the μ capture rate compared with present experimental limits. These values are calculated with the scale parameter ζz chosen at 400 TeV at the middle of the estimated range, the same value as that used earlier in [6] for calculating FCNC effects in meson mass differences and rare meson decays. For a discussion on the dependence of these results on the scale ζz , see text. The nuclear form factors were estimated using the dipole formula (14) except for *Pb* for which a more detailed and realistic model was used [22]. The more accurate formula (10) was used to estimate Q .

Pauli principle, namely that a proton which captures a muon must turn into a neutron and this it cannot do if the neutron state it would fill is already occupied by an existing neutron. The empirical values for this parameter δ have been determined in [21] and ranges in value from 3.4 to 3.2 for medium to heavy nuclei. With equations (11) and (15) we can write then the branching ratio as:

$$B_{\mu-e} = \frac{\Gamma_{conv.}}{\Gamma_{cap.}} = \frac{1/(\zeta z)^4}{G_F^2} \frac{Q^2/Z}{(g_V^2 + 3g_A^2)} |F(q^2)|^2 \frac{1}{(1 - \delta_{\frac{N}{2A}})}. \quad (16)$$

Finally, we turn to numerical results and their confrontation with experiment. In Table 1, we list the branching ratios for coherent muon-electron conversion in several nuclei for which stringent limits are either already available [2] or soon to be examined in planned experiments. The theoretical conversion rates are calculated in DSM using the formulae (11), (13), and (14), with the *effective charges* Z_{eff} taken from [17, 18, 19] and the nuclear form factor taken at the dominant kinematic point $q^2 \sim -m_\mu^2$. The scale parameter ζz is taken at 400 TeV in the middle of the range that we have estimated in the manner explained above, and the branching ratios are normalized to the experimentally measured values for $\Gamma_{cap.}$ [17] also listed.

In Table 1, one notes first that all the predicted branching ratios are roughly within the limits set by present experiment. This we find very gratifying because, all the parameters in the scheme having already been fixed by earlier work in other physical contexts, there is no freedom left in the calculation. A priori, no relation need exist linking to muon-electron conversion such widely different phenomena as quark masses, neutrino oscillations, meson decays, and cosmic ray air showers,

from which our parameters were determined. They are brought together in our case only through the DSM scheme. If the scheme is altogether false, there would be no reason why our predictions here for muon-electron conversion may not come out many orders of magnitude outside the experimental bounds, and yet the answers in Table 1 all turn out to fall neatly within. It is thus tempting to ascribe this coincidence to the DSM scheme's basic consistency. Secondly, one notices that the predicted values are all close to the experimental limit in all cases where this is known, and thus should very soon be accessible to detection in the next generation of experiments. Particularly interesting in this context are the cases of $^{27}\text{Al}_{13}$ [4] and $^{48}\text{Ti}_{22}$ [3] for which plans to increase the experimental sensitivity by 3 to 4 orders of magnitude are already underway.

It should be noted, however, that the predictions given in Table 1 for the muon-electron conversion rate are very sensitive to the chosen value of the scale parameter ζz , occurring as it does in (11) as the fourth power. We recall that the crucial upper bound on this parameter was deduced only through a somewhat qualitative argument from a suggested neutrino explanation for post-GZK air showers [14], and is thus unsure. Nevertheless, if experimental sensitivity is improved by 4 orders of magnitude as planned and still no muon-electron conversion in nuclei is seen, it would mean pushing up our estimate of the scale parameter ζz by another order of magnitude which would make the neutrino explanation for post-GZK air showers much less attractive, perhaps even untenable. This would be disappointing, for this suggestion is so far the only direct test one has for the assumption that generation is indeed dual color and not some other 3-fold horizontal gauge symmetry [11], although such an outcome would still not invalidate the basic tenets of the DSM scheme. On the other hand, if muon-electron conversion is observed with roughly the predicted rates, then one can imagine turning the argument around and use the consequent estimate for the scale ζz to make definite predictions for neutrino interactions at ultra-high energy, to be tested with post-GZK air showers, at the planned Auger project, for example. However, a more serious scenario to the DSM scheme proper than a failure to observe any muon-electron conversion at roughly the predicted rate in Table 1 would be if muon-electron conversion is observed but turns out to have very different relative rates for the different nuclei than those predicted, for this would cast doubt either on the way the nuclear physics is handled above, or else on the whole mixing pattern of the DSM scheme from which the quark CKM matrix and neutrino oscillation parameters were evaluated.

The phenomenon of muon-electron conversion can of course be considered also in contexts other than coherent reactions in nuclei. Another scenario in which it has been studied experimentally in some detail is in muonium-antimuonium conversion. Using similar arguments based on the DSM scheme as those detailed above, but tailored to the atomic environment in muonium, we have also made a study of the conversion rate there. The value of the effective coupling relevant

for this system in the DSM scheme is:

$$G_{DSM} \sim 2.5 \times 10^{-7} G_F,$$

which in turn implies a conversion probability for muonium-antimuonium conversion integrated over all times of the order:

$$P_{M\bar{M}} \sim 10^{-18}.$$

At present, the best experimental bounds are in the range of 10^{-10} [23], i.e. some 8 orders of magnitude above the predicted rates, which are thus presumably inaccessible for some time with the present experimental setup [24]. For this reason the details of this analysis will not be presented here.

In summary, we conclude that the DSM scheme, as at present parametrized and conceived, is consistent with the existing, already quite stringent, experimental bounds on muon-electron conversion, whether in nuclei or in muonium, and that the predicted rates in coherent nuclear reactions are such as to make it very likely for the phenomenon to be discovered in the new generation of experiments now being planned.

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